Benha University **Faculty of Engineering** **Shoubra**

Antennas &Wave Propagation

Electrical Eng. Dept. **4^{th} year communication** **2015-2016**

Sheet (3) - Solution

1. The maximum radiation intensity of a 90% efficiency antenna is 200 mW/ unit solid angle. Find the directivity and gain (dimensionless and in dB) when the
(a) Input power is 125.66 mW
(b) Radiated power is 125.66 mW

(a) $D_o = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi(200\times10^{-3})}{0.9(125.66\times10^{-3})} = 22.22 = 13.47dB$

$G_o = \epsilon_t \cdot D_o = 0.9(22.22) = 20 = 13.01dB$

(b) $D_o = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi(200\times10^{-3})}{(125.66\times10^{-3})} = 20 = 13.01dB$

$G_o = \epsilon_t \cdot D_o = 0.9\cdot(20) = 18 = 12.55dB$

2. 1GHz satellite antenna has an E-plane beam-width of 12^o and on H-plane beam-width of 10^o. The antenna conductivity and mismatch total loss -3db. Estimate the gain of antenna.

-3db=10Log(Losses) → $\eta = (1 - Losses)$ → $\eta=0.5$

$D\ approximate = \frac{41253}{\theta_{HP}\phi_{HP}} = \frac{41253}{10*12} = 343.8$

$G = \eta*D = 0.5*343.8 = 172.$ → $G_{db} = 10\log 172 = 22.4db.$

3. A lossless resonant half-wavelength dipole antenna, with input impedance of 73 ohms, is connected to a transmission line whose characteristic impedance is 50 ohms. Assuming that the pattern of the antenna is given approximately by $U=B_o\sin^3\theta$. Find the maximum gain and maximum absolute gain of this antenna.

$$U|_{max} = U_{max} = B_0$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi = 2\pi B_0 \int_0^{\pi} \sin^4\theta\, d\theta = B_0\left(\frac{3\pi^2}{4}\right)$$

$$D_0 = 4\pi\frac{U_{max}}{P_{rad}} = \frac{16}{3\pi} = 1.697$$

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Since the antenna was stated to be lossless, then the radiation efficiency $e_{cd} = 1$.

$$G_0 = e_{cd} D_0 = 1(1.697) = 1.697$$

$$e_r = (1 - |\Gamma|^2) = \left(1 - \left|\frac{73-50}{73+50}\right|^2\right) = 0.965$$

$$G_{0abs} = e_0 D_0 = 0.965(1.697) = 1.6376$$

4. Calculate the directivity of an antenna with circular aperture of diameter 3 meter at frequency 5 GHZ.

$$D = \frac{4\Pi}{\lambda^2} A_{em} = \frac{4\Pi}{\lambda^2} * (\Pi * r^2) = \frac{4\Pi}{(\frac{3*10^8}{5*10^9})^2} * (\Pi * (1.5)^2) = 24674.$$

5. If the aperture efficiency of an antenna is 0.7 and the beam traveling at 6 GHZ. Calculate the directivity, HPBW, and FNBW (approximately). Given circular aperture of diameter 3 meter.

$$D = \frac{4\Pi}{\lambda^2} * \eta * A_{em} = \frac{4\Pi}{\lambda^2} * 0.7 * (\Pi * r^2) = \frac{4\Pi}{(\frac{3*10^8}{6*10^9})^2} * 0.7 * (\Pi * (1.5)^2) = 24871$$

So D=24871.

$$D = \frac{41253}{\theta_{HP}\phi_{HP}} = \frac{41253}{(\theta_{HP})^2} = 24871$$

So $(\theta_{HP}) = 1.28^o$.

FNBW $= 2*(\theta_{HP}) = 2.57^o$.

6. What is the maximum effective aperture (approximately) for a beam antenna having HPBW of 30º & 35º in perpendicular planes intersecting in the beam axis? Minor lobes are small and may be neglected.

$$D = \frac{4\Pi}{\lambda^2} A_{em} \rightarrow A_{em} = \frac{D}{4\Pi}\lambda^2$$

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$$D = \frac{41253}{\theta_{HP}\phi_{HP}} = \frac{41253}{30*35} = 39.3$$

$$A_{em} = \frac{D}{4\Pi}\lambda^2 = 3.2\,\lambda^2$$

7. An antenna has a uniform field pattern for θ between (45°&90°) ، φ between (0° & 120°), if E=3V/m at a distance of 500m from the antenna & maximum current is 5A, find the radiation resistance of antenna, Directivity, and effective aperture?

$$P = SA = 0.5*I^2R_r = 0.5*\frac{E^2}{Z}\int_{\theta=\frac{\Pi}{4}}^{\frac{\Pi}{2}}\int_{\phi=0}^{\frac{2\Pi}{3}} r^2 Sin\theta d\theta d\varphi = 0.5*(5)^2R_r \rightarrow$$

$R_r=281\Omega$.

$$\Omega_A = \int_{\theta=\frac{\Pi}{4}}^{\frac{\Pi}{2}}\int_{\varphi=0}^{\frac{2\Pi}{3}} Sin\theta d\theta d\varphi = 1.18$$

$$A_{em} = \frac{\lambda^2}{\Omega_A} = 0.85\lambda^2.$$

$$D = \frac{4\Pi}{\Omega_A} = 10.67.$$

8. An isotropic antenna has a field pattern given by E=10 I_o /r V/m, where I is the amplitude of current, r is distance (m), find R_r. repeat for hemisphere antenna.

$P = SA = \frac{E^2}{Z}A = 0.5\frac{100I^2}{r^2*Z}(4\Pi*r^2)$ → for hemisphere $A=(2\Pi*r^2)$

$P = 0.5I_o^{\,2}R_r$

So $0.5\frac{100I_o^{\,2}}{r^2*Z}(4\Pi*r^2) = 0.5I_o^{\,2}R_r \rightarrow R_r = 3.33\Omega$

9. Find R_r of a unidirectional pattern of antenna with $U=8Sin^2\theta\, Sin^3\varphi$ wsr^{-1}, where $0\le\theta\le\Pi$ & $0\le\varphi\le\Pi$. If I_{rms}=3A.

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$P_{rad}=\int_0^{\Pi}\int_0^{\Pi} U d\Omega=\int_0^{\Pi}\int_0^{\Pi}(8Sin^2\theta Sin^3\phi)*Sin\theta d\theta d\phi = I^2 R_r$

$R_r = 1.6\ \Omega$

10. What is the amplitude of current that would be required in a short dipole of length 0.05λ to produce 100w of radiated power? Assume that the medium surrounding the short dipole in air and the current is uniform distribution.

$P_{rad} = \frac{1}{2} I_o^2 R_r = 100$ *(note: I_o^2 .. amplitude or max. current.. not terminal)*

$R_r = 80\Pi^2 \frac{L^2}{\lambda^2} = 80\Pi^2 \frac{(0.05\lambda)^2}{\lambda^2} = 1.97\Omega$

So $100 = \frac{1}{2} I_o^2 * 1.97 \rightarrow I_o = 10A.$

Note: for $\frac{\lambda}{2}$ *dipole(half wave dipole) ...* $R_r = 73\Omega$

11. What is the max? Power received at a distance of 0.5 Km. over a free-space 1GHZ circuit consisting of a transmitting antenna with 25dB gain and receiving antenna with 20dB gain? The gain is with respect to a lossless isotropic source. The transmitting antenna input is 150W.

$\lambda = \frac{C}{F} = 3*10^8/1*10^9 = 0.3m.$

$\frac{P_r}{P_{in}} = G_{in} G_r \left(\frac{\lambda}{4\Pi R}\right)^2.$

$G_r|_{db} = 10\log G_r \rightarrow G_r = 100$

$G_{in}|_{db} = 10\log G_{in} \rightarrow G_{in} = 316.22$

$\frac{P_r}{150} = 100 * 316.22\left(\frac{0.3}{4\Pi * 0.5 * 10^3}\right)^2 \rightarrow P_r = 10.8mw.$

12. A wave traveling normally outward from the page (toward the reader) is the resultant of two elliptically polarized waves, one with components of E given by:

$$\mathscr{E}'_y = 3\cos\omega t$$

$$\mathscr{E}'_x = 7\cos\left(\omega t + \frac{\pi}{2}\right)$$

And the other with components given by:

$$\mathscr{E}''_y = 2\cos\omega t$$

$$\mathscr{E}''_x = 3\cos\left(\omega t - \frac{\pi}{2}\right)$$

(a) What is the axial ratio of the resultant wave?
(b) Does the resultant vector E rotate clockwise or counterclockwise?

(a) $E_y = E'_y + E''_y = 3\cos\omega t + 2\cos\omega t = 5\cos\omega t$

$E_x = E'_x + E''_x = 7\cos(\omega t + \frac{\pi}{2}) + 3\cos(\omega t - \frac{\pi}{2})$

$= -7\sin\omega t + 3\sin\omega t = -4\sin\omega t$

$AR = \frac{5}{4} = 1.25$

(b) At $\omega t = 0$, $\vec{E} = 5\hat{a}_y$

At $\omega t = \pi/2 \Rightarrow \vec{E} = -4\hat{a}_x \Rightarrow$ Rotation in CCW

13. A wave traveling normally out of the page is resultant two elliptically polarized (EP) waves, one with components $E_x = 5\cos\omega t$ and $E_y = 3\sin\omega t$ and another with components $E_r = 3e^{j\omega t}$ and $E_L = 4e^{-j\omega t}$. For the resultant wave, find (a) AR, and (b) the band of rotation and polarization.

1st component
$E_{x1} = 5\cos\omega t$
$E_{y1} = 3\sin\omega t.$
2nd component
$E_r = 3e^{j\omega t} = 3\cos\omega t + j3\sin\omega t$
$E_l = 4e^{-j\omega t} = 4\cos\omega t - j4\sin\omega t$
So $E_{x2} = 3\cos\omega t + 4\cos\omega t = 7\cos\omega t$

E_{y2}=3Sinωt-4Sinωt=-Sinωt

Total components

$E_{xt}= E_{x1} + E_{x2}=5\cos\omega t +7\cos\omega t=12 \cos\omega t$.

$E_{yt}=E_{y1}+ E_{y2}=3\sin\omega t - \sin\omega t=2\sin\omega t$

So $(\frac{E_{xt}}{12})^2 + (\frac{E_{yt}}{2})^2 = 1$... *Ellipse*

(a) *AR = 12/2=6*

(b) *Put ωt=0 , 90 , you will find that this wave is Right polarized & CCW*

REPORT

1. Design an antenna with omnidirectional amplitude pattern with a half-power beam width of 90°, Express its radiation intensity by $U=\sin^n\theta$. Determine the value of n and attempt to identify elements that exhibit such a pattern. Determine the directivity of the antenna.

Solution: Since the half-power beamwidth is 90°, the angle at which the half-power point occurs is $\theta = 45°$. Thus

$$U(\theta = 45°) = 0.5 = \sin^n(45°) = (0.707)^n$$

or

$$n = 2$$

$$U_{max} = 1$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} \sin^2\theta \, \sin\theta \, d\theta \, d\phi = \frac{8\pi}{3}$$

$$D_0 = \frac{4\pi}{8\pi/3} = \frac{3}{2} = 1.761 \text{ dB}$$

2. A uniform plane wave, of is traveling in the positive z-direction. Find the polarization (linear, circular, or elliptical), sense of rotation (CW or CCW), when
(a) Ex =Ey, Δφ=φy−φx =0 (b) Ex ≠Ey, Δφ=φy−φx =0
(c) Ex =Ey, Δφ=φy−φx =π/2 (d) Ex =Ey, Δφ=φy−φx =−π/2

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(e) Ex =Ey, Δφ=φy−φx =π/4 (f) Ex =Ey, Δφ=φy−φx =−π/4
(g) Ex =0.5Ey, Δφ=φy−φx =π/2 (h) Ex =0.5Ey, Δφ=φy−φx =−π/2

(a) Linear because $\Delta\phi = 0$.
(b) Linear because $\Delta\phi = 0$.
(c) Circular because 1. $E_x = E_y$
2. $\Delta\phi = \pi/2$ CCW
(d) Circular because 1. $E_x = E_y$
2. $\Delta\phi = -\pi/2$ CW
(e) Elliptical because $\Delta\phi$ is not multiples of $\pi/2$. CCW
(f) Elliptical because $\Delta\phi$ is not multiples of $\pi/2$ CW
(g). Elliptical because 1. $E_x \neq E_y$
2. $\Delta\phi$ is not zero or multiples of π.
CCW
(h) Elliptical because 1. $E_x \neq E_y$
2. $\Delta\phi$ is not zero or multiples of π.
CW

3. Calculate the polarization loss factor (PLF)…in db and dimensionless of an antenna whose electric field polarization is expressed as: $\vec{E}_a = (a\hat{x} + a\hat{y})E(r,\theta,\phi)$, when the electric field of the incident wave given by $\vec{E}_i = a\hat{x}E_o(x,y)e^{-jkz}$.

- *Unit vector of* $\vec{E}_a = \hat{P}_a = \dfrac{a\hat{x} + a\hat{y}}{\sqrt{1+1}}$
- *Unit vector of* $\vec{E}_i = \hat{P}_w = \dfrac{a\hat{x}}{\sqrt{1}}$
- *PLF* = $|\hat{P}_w \cdot \hat{P}_a|^2$ *=1/2.*
- *PLF*$|_{db}$ *= 10log(0.5)=-3db*

Good Luck

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